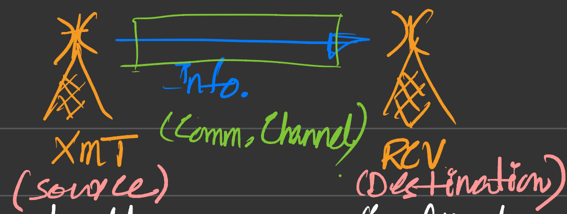
Signal Analysis & Communication ECE 355

Ch. 8.1. Modulation.

Lecture 32.

29-11-2023

Ch. 8. COMMUNICATION SYSTEMS (CS)



- Generally, in all CSs, the information at the source is first processed by a transmitter/modulator to change it into a form suitable for transmission over the communication channel.

- At the receiver, the sig. is then recovered through appropriate processing.

- Example

- The atmosphere will rapidly attenuate (weakens sig. strength) sig. in the audible freq. range (10 Hz → 20 kHz), whereas it will propagate signals at a higher freq. range over long distances.

- Therefore, to transmit audio sig. over a comm. ch. that relies on propagation through the atmosphere, the transmitter embeds the sig. to higher freq. sig.

  ↓ This process is called "Modulation"

- The modulated sig. is then transmitted through the atmosphere, which is then received at the receiver by extracting the info. bearing sig. through the process of "Demodulation".

- That's the major reason of applying MODULATION, i.e.,

① To embed information into high freq. sig.'s that can be transmitted effectively.

NOTE: - If we donot modulate, we would need impractically larger antenna size for longer distance transmission.

- Antenna size is normally $\frac{\lambda}{10} \rightarrow \frac{\lambda}{2}$

where λ is the wavelength of the sig. $\lambda = \frac{c}{f}$

- For the sig. with $f = 10\ kHz$

$$\lambda = \frac{3 \times 10^8}{10 \times 10^3} = 3 \times 10^4$$

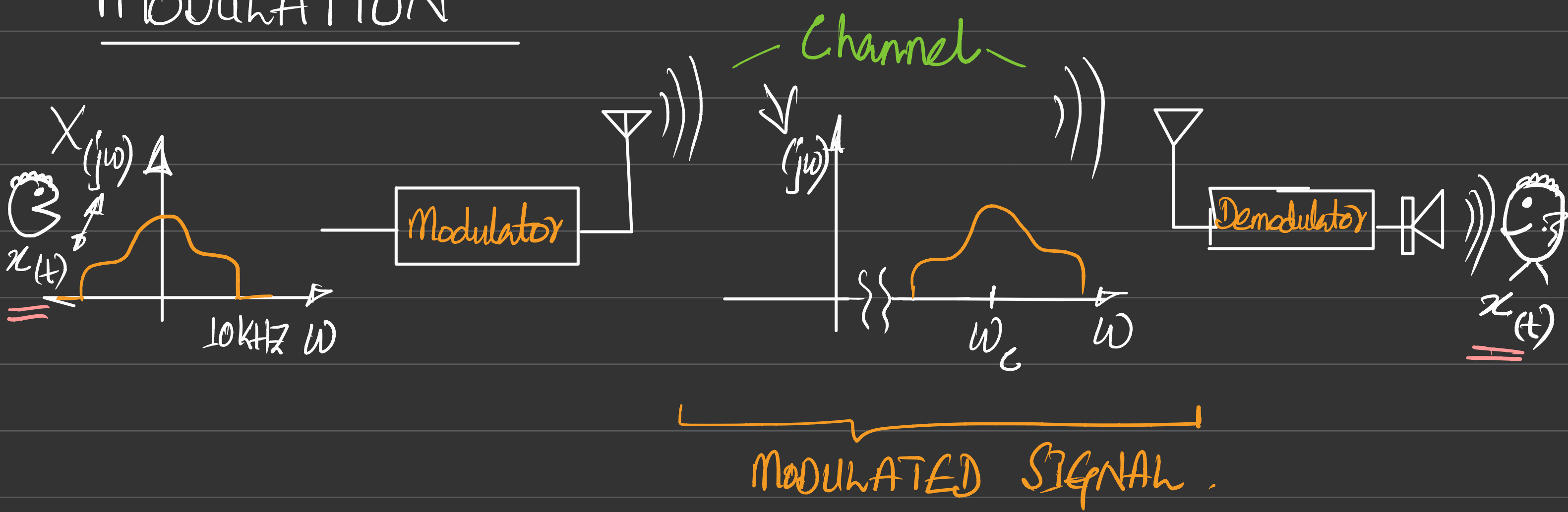
$\therefore$ Reqd. antenna size: $3 \times 10^3 \longrightarrow 1.5 \times 10^4\ m$.

(IMPRACTICAL!)

- Another reason is:

② To make possible the simultaneous transmission of more than one sig. with overlapping spectra over the same channel, through "MULTIPLEXING".

MODULATION



Convention:

$x_{(t)}$: "Baseband signal" / "Modulating signal"

w_c: "Carrier Frequency"
e.g. broadcast radio / TV ~ MHz.
WiFi: 2.4 / 5 GHz.
Microwave / Satellite: 1 ~ 300 GHz.

$y_{(t)}$: "Passband signal" / "Modulated signal"

Ch. 8.1. AMPLITUDE MODULATION (Am)

- A complex exponential or sinusoidal sig., called carrier sig. with freq. suitable for transmission over the communication channel, has its amplitude multiplied (modulated) by the information bearing sig. $x_{(t)}$.

$$y_{(t)} = x_{(t)} c_{(t)}$$

carrier sig.

Am with Complex Exponential Carrier:

- The carrier is a complex exponential of the form:

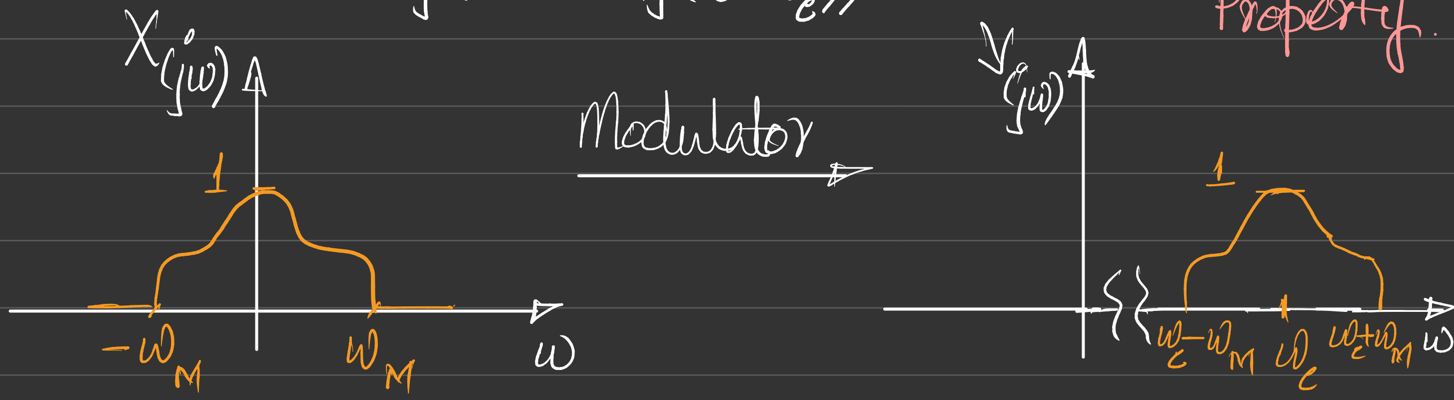
$$c_{(t)} = e^{j(w_c t + \theta_c)} \quad — (1)$$

- For convenience, let $\theta_c = 0$ (ideal)

$$y_{(t)} = x_{(t)} e^{j w_c t}$$

$$Y(j\omega) = X(j(\omega - \omega_c))$$

Freq. shift Property.

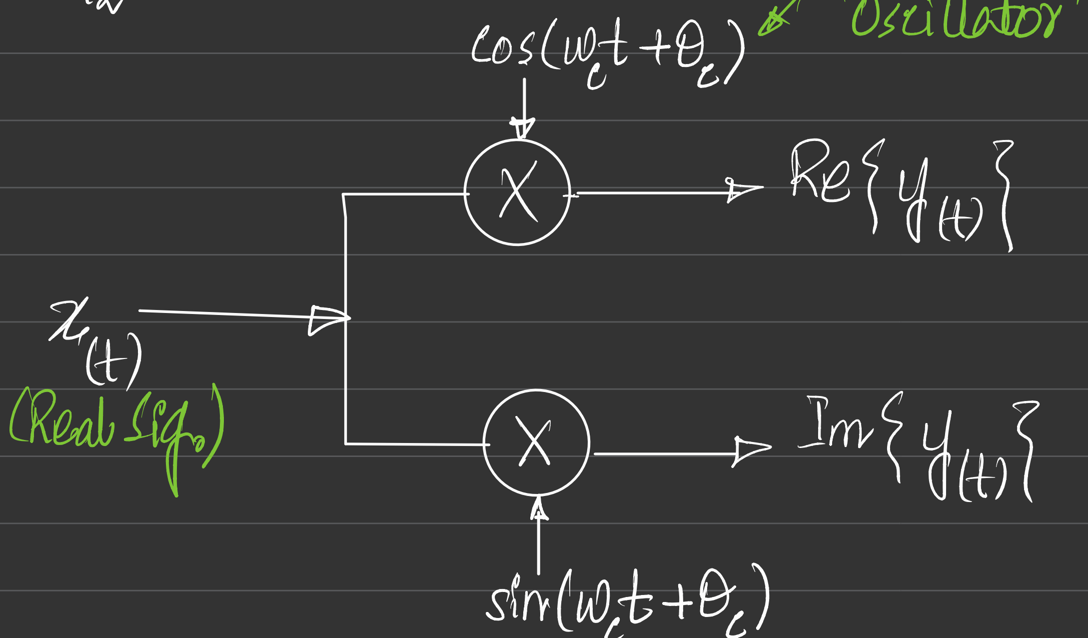


- Since the carrier, $c(t)$, is a complex sig., $y(t)$ can be written as:

$$\text{eqn } ① \Rightarrow y(t) = x(t)\cos\omega_c t + j\,x(t)\sin\omega_c t$$

$$= Re\{y(t)\} + Im\{y(t)\}$$

- Therefore, the implementation of eqn ① utilizes two separate multipliers & two sinusoidal carrier signals that have a phase difference of $\pi/2$ (cos & sin).



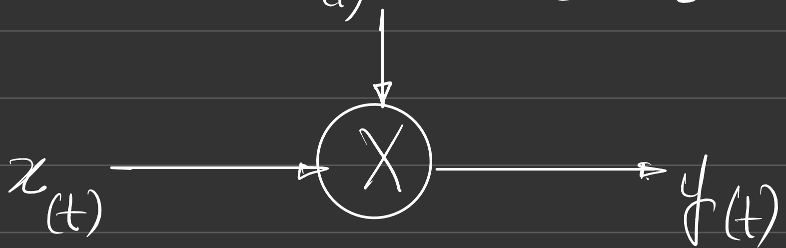
AM with Sinusoidal Carrier

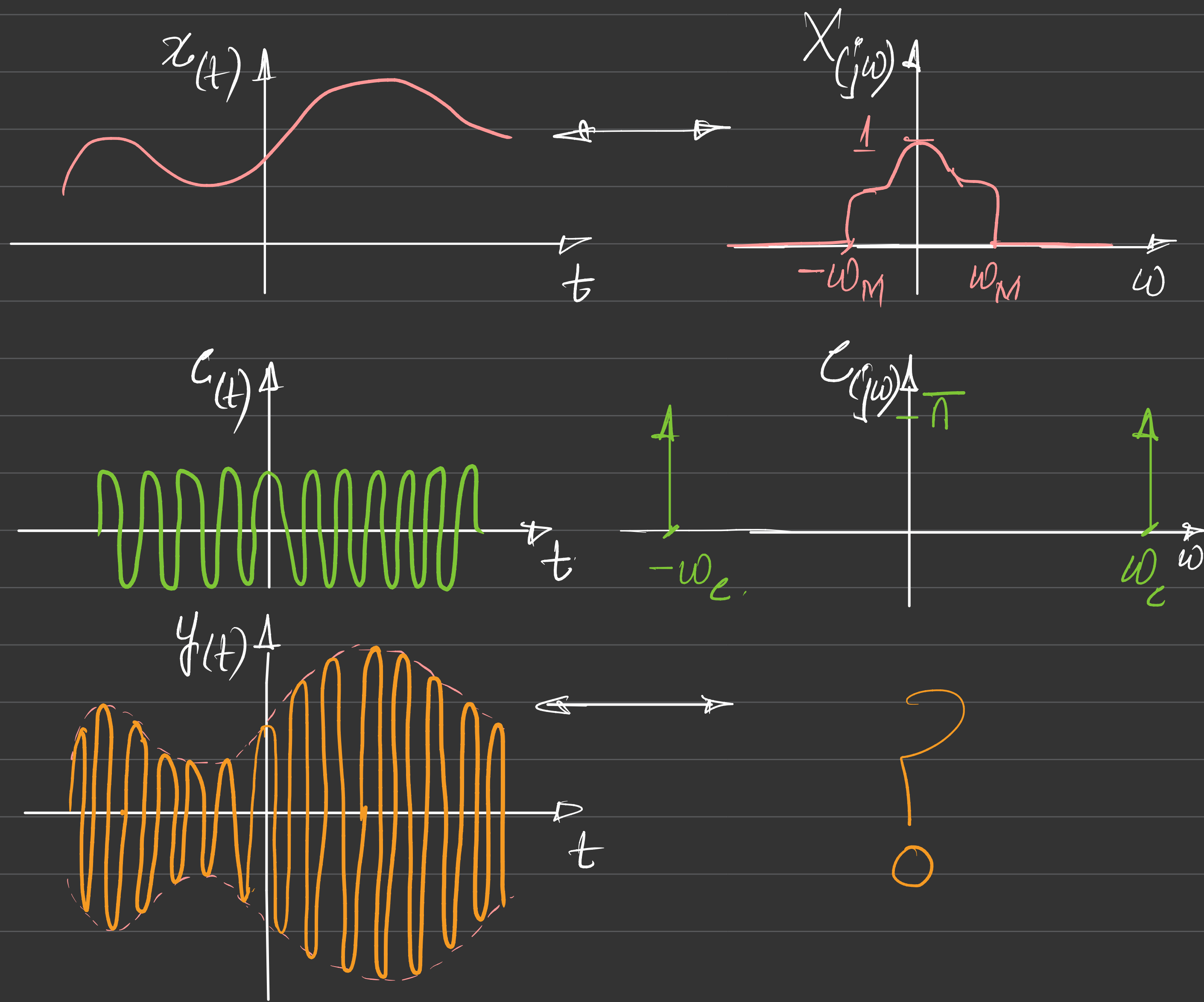
- for many cases, using a sinusoidal carrier is often simpler & equally effective as using complex exponential carrier.

$$c(t) = \cos(\omega_c t + \theta_c) \quad \text{--- (2)}$$

retaining only real or imaginary part.

$$c(t) = \cos(\omega_c t + \theta_c)$$





- Assuming, $\theta_c = 0$ (for simplicity/ideal), CTFT of eqn ② is:

$$C_{(j\omega)} = \pi \left[\delta_{(\omega-\omega_c)} + \delta_{(\omega+\omega_c)} \right]$$

* $\cos(\omega_c t) = \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$

* $e^{j\omega_c t} = 2\pi \delta_{(\omega-\omega_c)}$

- For $Y_{(j\omega)}$:

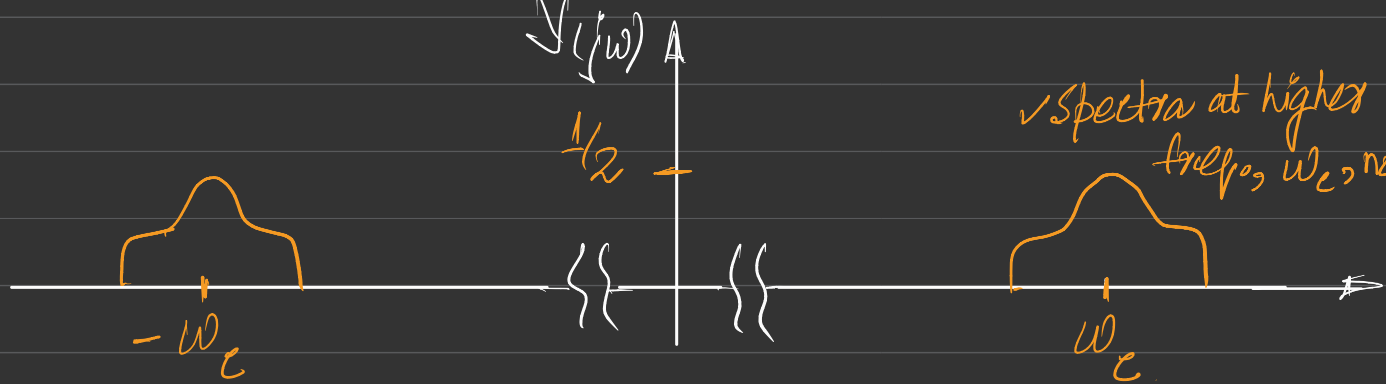
$$y_{(t)} = x_{(t)} c_{(t)}$$

$$Y_{(j\omega)} = X_{(j\omega)} * C_{(j\omega)}$$

$$Y_{(j\omega)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_{(j\theta)} C_{(j(\omega-\theta))} d\theta$$

$$= \frac{1}{2} \left[X_{(j(\omega-\omega_c))} + X_{(j(\omega+\omega_c))} \right]$$

Using sampling property of impulse.



✓ spectra at higher freq., ω_c, now.

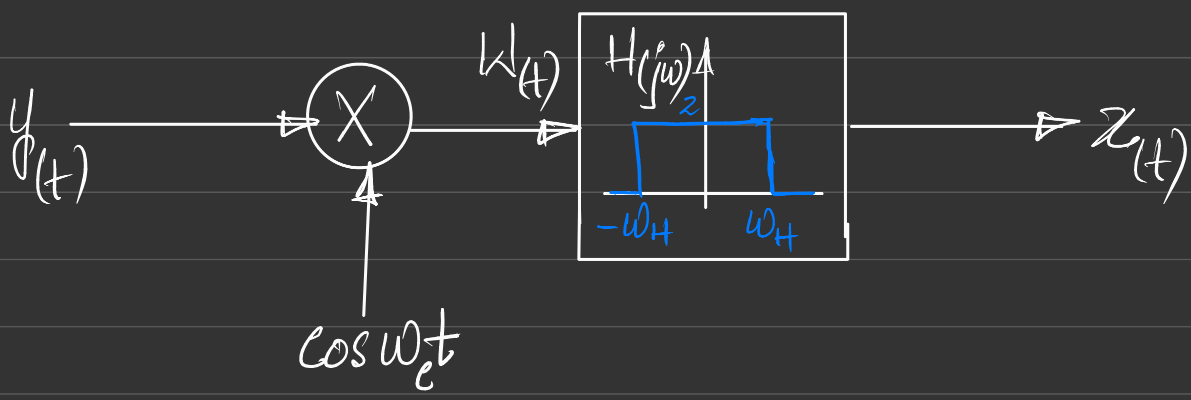
DEMODULATION (ideal)

- To recover the sig. at the receiver.
- If Am using the carrier given in eqn ①, we want

$$x_{(t)} = y_{(t)} e^{-j\omega_c t} \quad — ③$$

- In freq. domain, this has the effect of shifting the spectrum of the modulated sig. back to its original position on the freq. axis.

- Considering AM using sinusoidal carrier as in eqn ②, to demodulate:



- "Synchronous Demodulation" as we assume $\theta_c = 0$ in both transmitter & receiver.

- In time-domain:

$$y_{(t)} = x_{(t)} \cos \omega_c t \qquad \theta_c = 0$$

$$w_{(t)} = y_{(t)} \cos \omega_c t \qquad \theta_c = 0$$

$$= x_{(t)} \cos^2(\omega_c t) \qquad - ④$$

$$= x_{(t)} \left(\frac{1}{2} + \frac{1}{2} \cos(2\omega_c t) \right)$$

$$= \underbrace{\frac{1}{2} x_{(t)}}_{\text{Pass this part using LPF with gain 2.}} + \cancel{\frac{1}{2} x_{(t)} \cos(2\omega_c t)}$$

High freq. part filtered out.

- In freq. domain

eqn 4 $\longleftrightarrow$ $$W(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j\theta)\, C(j(\omega-\theta))\, d\theta$$

$$= \frac{1}{2}\left[Y(j(\omega-\omega_c)) + Y(j(\omega+\omega_c))\right]$$