Signal Analysis & Communication ECE355

Ch. 7.1. Sampling

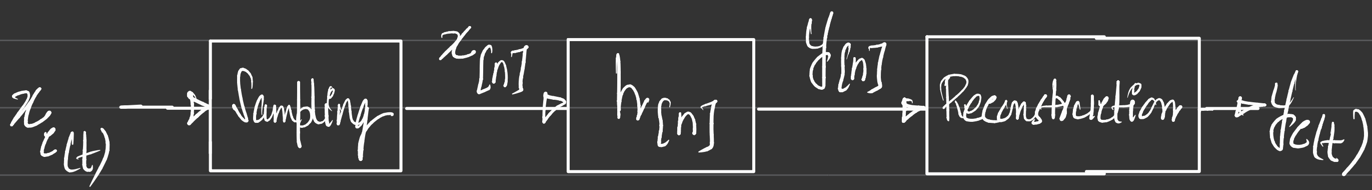
Lecture 28

20-11-2023

SAMPLING, PROCESSING, RECONSTRUCTION

Introduction

- In this topic, we will see the fact that under certain conditions (sampling theorem) a CT sig. can be completely recovered from a sequence of its samples (DT version)
- In many contexts, processing DT signals is more flexible & is often preferable to processing CT signals.
- This is due to the dramatic development of digital technology over the past few decades, resulting in the availability of inexpensive, light-weight, programmable, & easily reproducible DT systems.

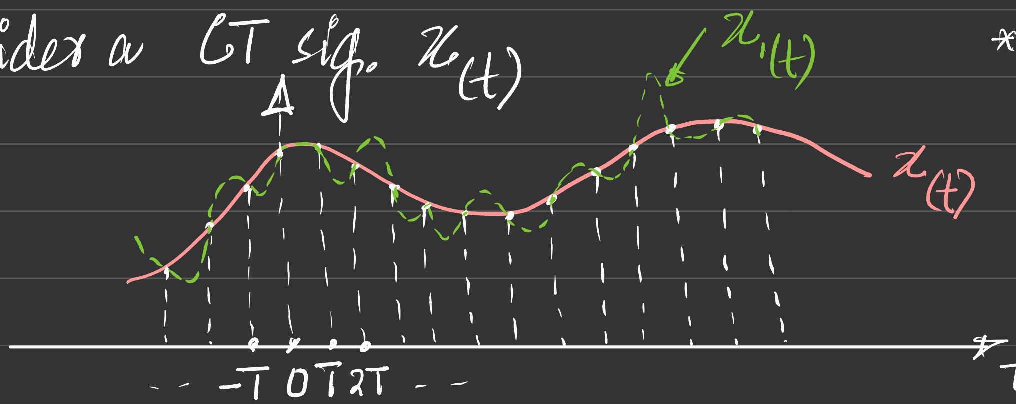


- Examples

1. Music : Autotune.
2. Car : Steering

Ch. 7.1. SAMPLING

- Consider a CT sig. $x(t)$



* there can be ∞ number of signals, which can generate same samples, e.g., $x_1(t)$.

- We sample the sig. at each integral multiple of $x(t)$.

$\Rightarrow$ Samples of $x(t)$: $\{x(nT)\}$, $n = 0, \pm 1, \pm 2, \ldots$

- Sampling Period: T
- Sampling Freq.: $\omega_s \triangleq \frac{2\pi}{T}$
- In general, $\{x(nT)\}$ does not uniquely determine $x(t)$ as shown in fig. !!!
- In order to develope the sampling theorem, we need a convenient way to represent the sampling of a CT sig. at regular intervals as follows:

IMPULSE TRAIN SAMPLING

- Periodic impulse train is multiplied by the CT sig., $x(t)$, that we wish to sample.
- Sampling Function: $p(t) \triangleq \sum_{n=-\infty}^{\infty} \delta(t - nT)$ (Periodic) "Impulse Train"
- Sampled sig. $x_p(t) \triangleq x(t)\, p(t)$ — (1)

$$= \sum_{n=-\infty}^{\infty} x(t)\, \delta(t - nT)$$

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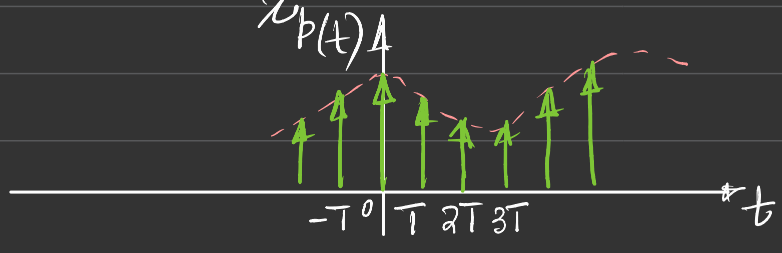


- Now, let's look at its spectrum.
- Using multiplication property of CTFT to eqn (1)

$$X_{p}(j\omega) = \frac{1}{2\pi}\left[X(j\omega) * P(j\omega)\right] \quad -(2)$$

- $P(j\omega)$ here is given as

$$p(t) \overset{\mathcal{F}}{\longleftrightarrow} P(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega - k\omega_s) \quad -(3)$$

$a_k = \frac{1}{T}$
CTFT of CT periodic sig.

- Inserting eqn (3) in eqn (2)

$$X_p(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\, P(j(\omega-\theta))\, d\theta$$

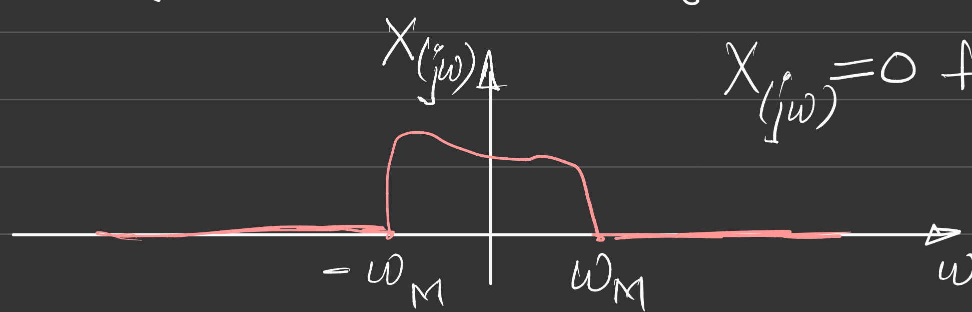
$$= \frac{1}{\cancel{2\pi}}\int_{-\infty}^{\infty} X(j\theta) \times \frac{\cancel{2\pi}}{T}\sum_{k=-\infty}^{\infty}\underline{\delta(\omega - k\omega_s - \theta)}\, d\theta$$

exists at $\theta = \omega - k\omega_s$

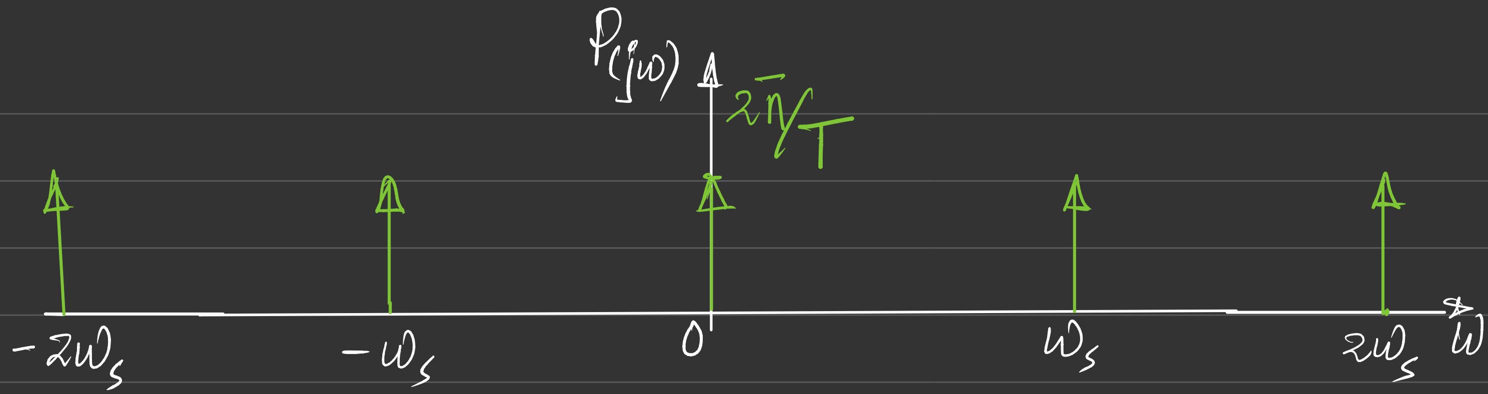
$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s)) \quad (1)$$

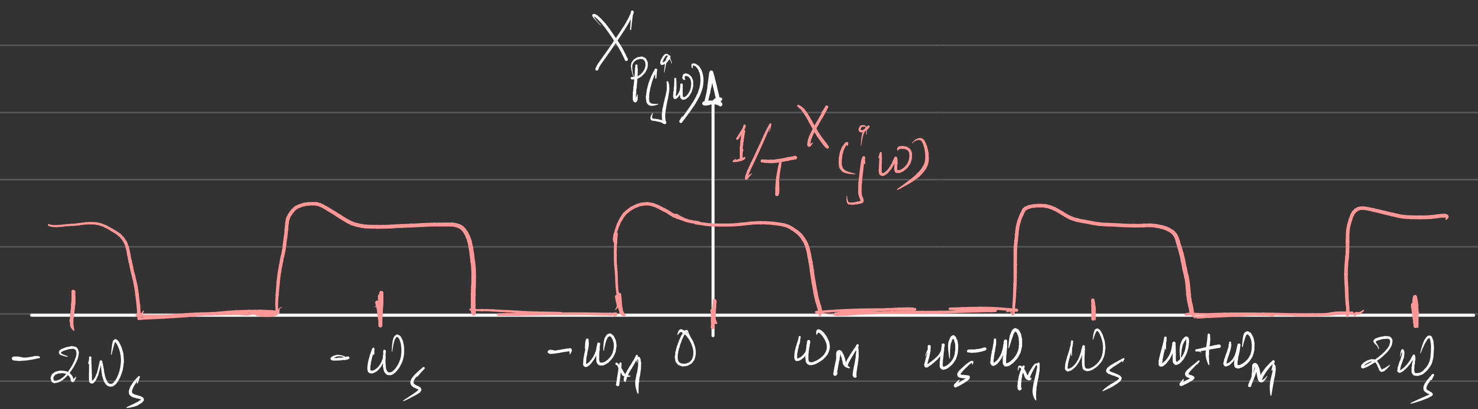
$\Rightarrow$ $X_{p}(j\omega)$ is a periodic function of ω consisting of superposition of shifted replicas of $X(j\omega)$, scaled by $1/T$.

- Suppose $X(j\omega)$ is <u>band-limited by ω_M</u>.

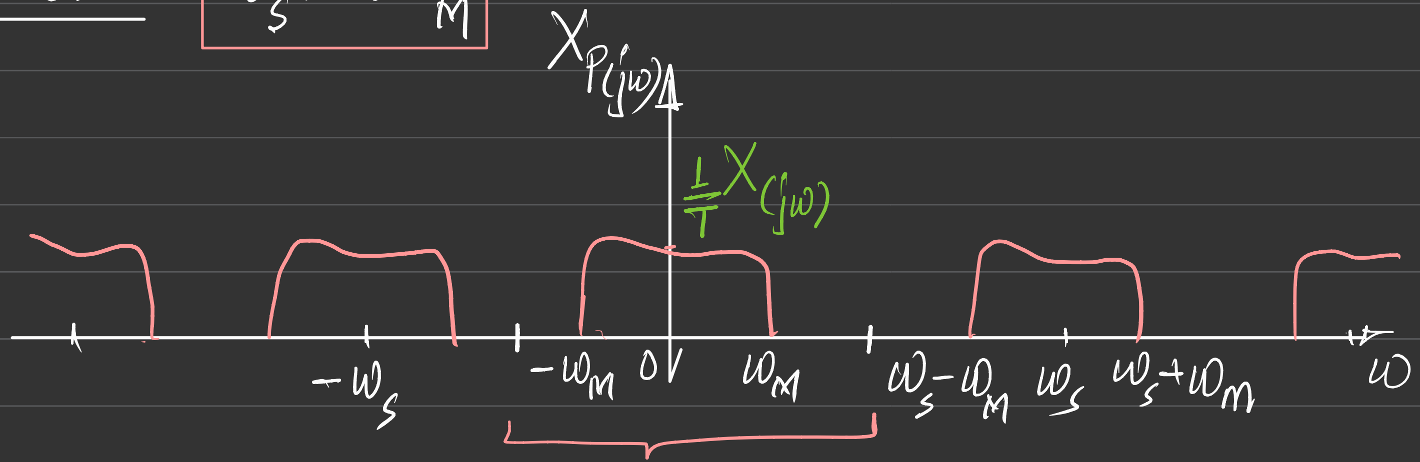


$X(j\omega) = 0$ for $|\omega| > \omega_M$





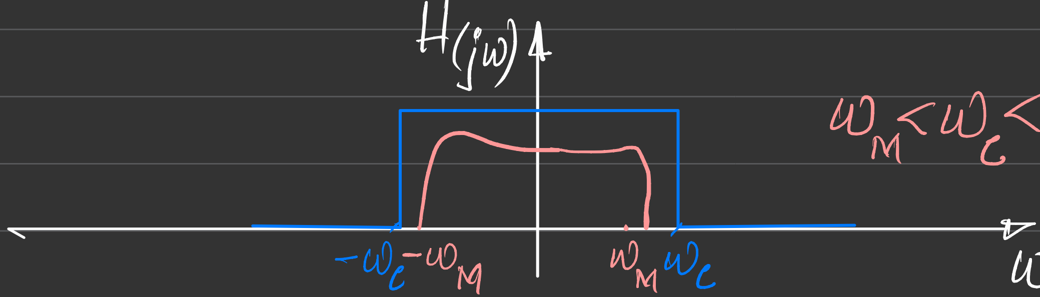
CASE $\boxed{\omega_s > 2\omega_M}$



No overlapping with the adjacent spectrums.

With $\omega_s < 2\omega_M$ there will be overlapping

- And we use ideal LPF to recover $x(t)$ from $x_p(t)$



$\omega_M < \omega_c < \omega_s - \omega_M$

Usually $\omega_c = \frac{\omega_s}{2}$

Nyquist Sampling Theorem

- Let $x(t)$ be band limited sig. such that $X(j\omega) = 0$ for $|\omega| > \omega_M$.

- It is uniquely determined by its samples $x(nT)$, $n = 0, \pm 1, \pm 2, \ldots$, if $\omega_s > 2\omega_M$, where $\omega_s = \frac{2\pi}{T}$.

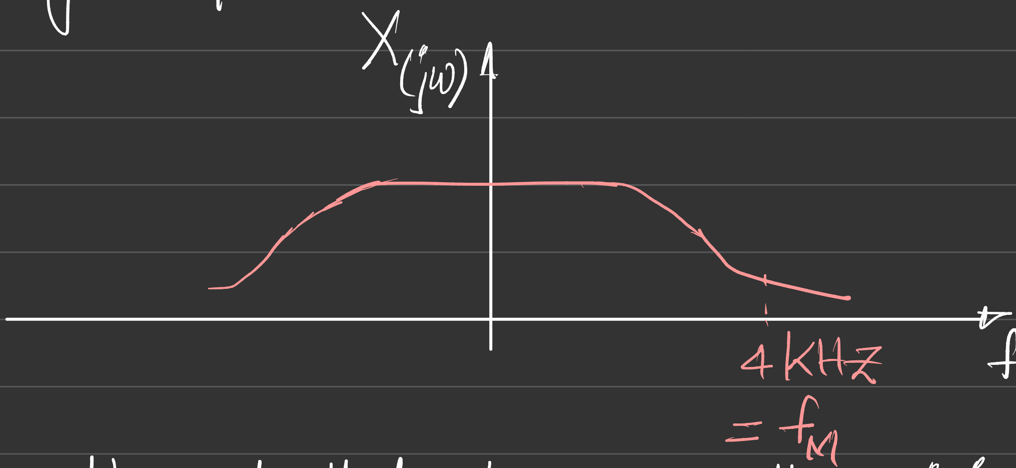
Nyquist Rate: $2\omega_M$

In Hertz: $f_M \triangleq \frac{\omega_M}{2\pi}$, $f_s = \frac{1}{T} > 2f_M$

"Sampling Rate"

Example

- Voice Signal Spectrum.



- To sample such that to recover the original sig.

$$f_s = \frac{1}{T} > 2 \times f_M$$

$$> 2 \times 4\,\text{kHz} = 8\,\text{KHz}.$$

- ∴ In 'Digital Telephony', the sampling rate is 8 KHz
- Human hearing range: 20 Hz to 20 KHz → accordingly, WAVE/CD uses 44100 samples/sec