# Signal Analysis & Communication ECE 355

## Ch. 5.1 : Discrete Time Fourier Transform (DTFT)

Lecture 23_02

01 - 11 - 2023

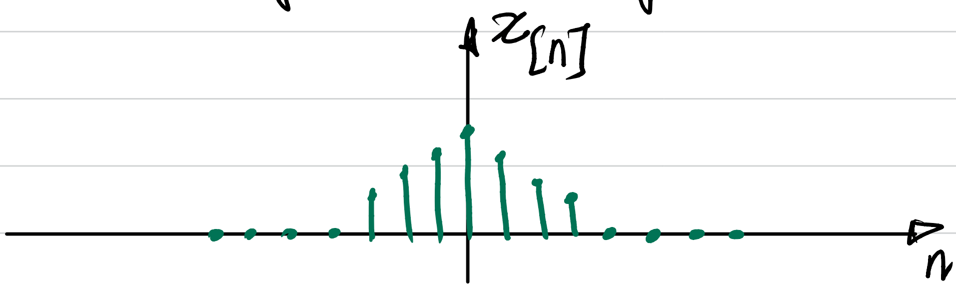
# Ch. 5.1: DISCRETE TIME FOURIER TRANSFORM

- Recall

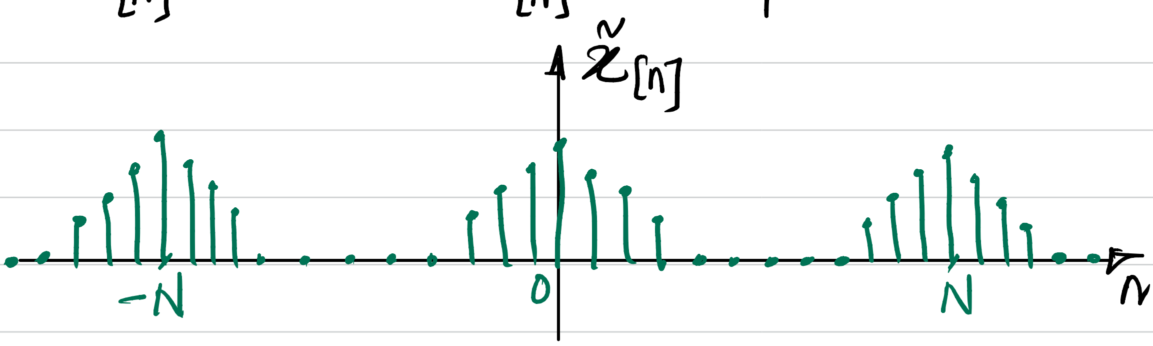
- The FS representation of a DT periodic sig. is a finite series as opposed to the infinite series representation of a CT Periodic signals.

* - There are corresponding differences b/w CT & DT Fourier Transforms.
- The transform pairs of the equations can easily be derived by extending the FS description of DT Periodic signals to DT Aperiodic signals as done for CT case.

- Consider the following DT Aperiodic sig.




- From this DT Aperiodic sig., we can construct a periodic sig. $\tilde{x}[n]$ for which $x[n]$ is one period as shown:




$$\tilde{x}[n] \longrightarrow x[n] \quad \text{as} \quad N \to \infty \qquad (k\omega_0 \to \omega)$$

- Working along the same lines as moving from CTFS to CTFT, we can derive DTFT from DTFS as under:

## DTFT:

$e^{j\omega}$ shows that DT is periodic in freq.-domain with period $2\pi$.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad \text{— (VI)}$$ ANALYSIS

Aperiodic in time-domain

## DTIFT:

* $$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega \quad \text{— (VII)}$$ SYNTHESIS

periodic in freq.-domain with period $2\pi$
(Discrete in time-domain)
$e^{j\omega n} = e^{j(\omega+2\pi)n}$

## Recall

## CTFT:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$ ANALYSIS.

## CTIFT:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$ SYNTHESIS.

## NOTE:

1. We can use $e^{j\omega} = z$ in eqn (VI)

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$ z-transform. (used in Control Sys.)

2. $X(e^{j\omega})$ is periodic with $2\pi$.

3. Convergence Conditions.
   - No issues in 'Synthesis' as integration is over a finite interval.
   - The 'Analysis' will converge if

$$\sum_{n=-\infty}^{\infty} |x_{[n]}| < \infty$$

OR if

$$\sum_{n=-\infty}^{\infty} |x_{[n]}|^2 < \infty$$

## Example 1

$$x_{[n]} = a^n u_{[n]} \quad , \quad |a| < 1$$

$*$ Similar to $x_{(t)} = e^{-at} u_{(t)}$

$$X_{(e^{j\omega})} = ?$$

$$X_{(e^{j\omega})} = \sum_{n=-\infty}^{\infty} a^n u_{[n]} e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (a e^{-j\omega})^n$$

$$\sum_{k=0}^{\infty} r^k \overset{|r|<1}{=} \frac{1}{1-r}$$

$$\overset{|a|<1}{=} \frac{1}{1-ae^{-j\omega}}$$

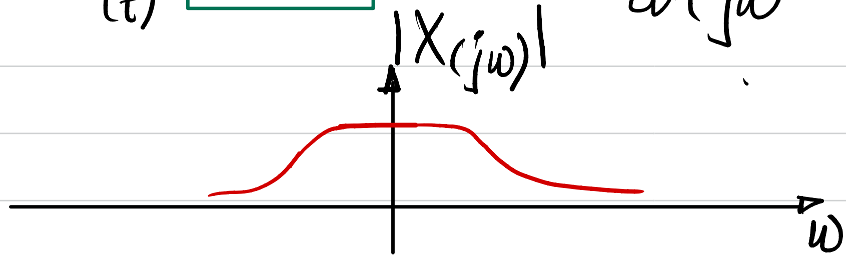
$$\left( = \frac{1}{1-a(\cos\omega - j\sin\omega)} \right)$$

$$= \frac{1}{\underbrace{(1-a\cos\omega)}_{Re.} + j\underbrace{a\sin\omega}_{Im.}}$$

— VIII

Compare it with:

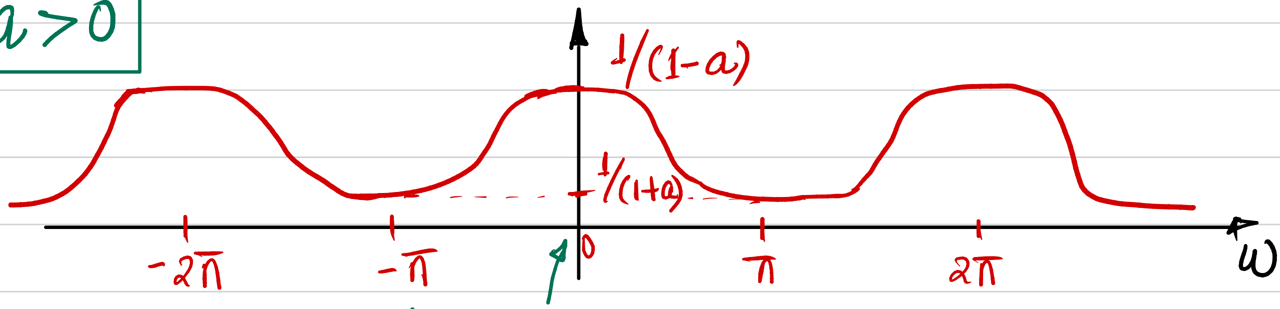
$$\left[ \; e^{-at} u_{(t)} \; \boxed{(a>0)} \; \overset{\mathcal{F}}{\longleftrightarrow} \; \frac{1}{a+j\omega} \; \right]$$



- Here for DT Aperiodic, eqn (VIII) $\Rightarrow$

$$|X(e^{j\omega})| = \frac{1}{\sqrt{1 + a^2 - 2a\cos\omega}}$$

$\boxed{1 > a > 0}$

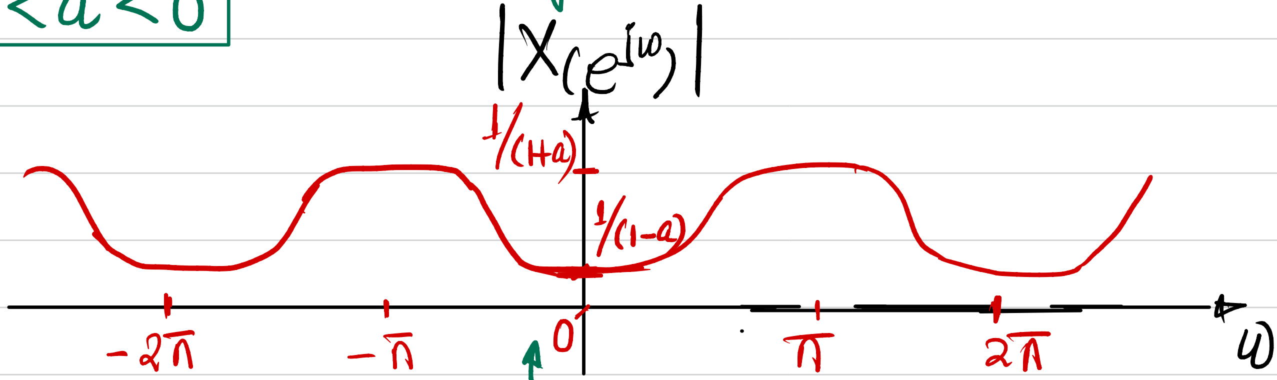



low freq.

high freq.

"Low Pass Sig./Filter"

$\boxed{-1 < a < 0}$




low freq.

high freq.

"High Pass Sig./Filter"